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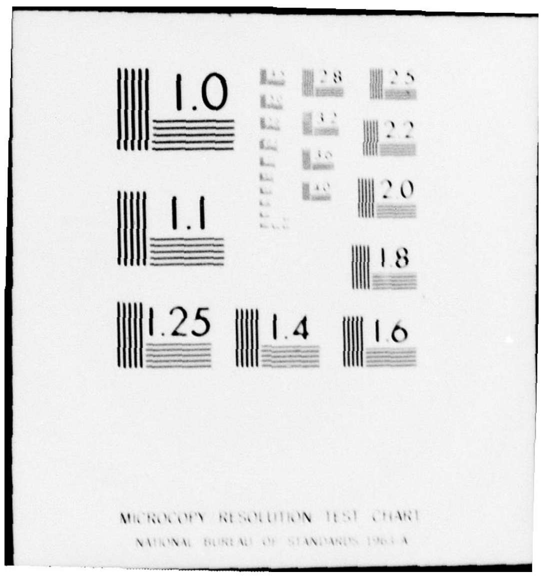
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# MATHEMATICS IN THE FIELD OF HISTORY.


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18 ARO

19 12494.11-M

9 Technical Report No. 14 USCEE-RB77-3



12 13p.

11 April 1977




Supported by the U. S. Department of the Army, Army Research Office, under Grant No. DAAG29-76-G-0027

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## MATHEMATICS IN THE FIELD OF HISTORY

### 1. Introduction

Galileo asserted that the language of science was mathematics, a dictum well substantiated over the intervening years. The outstanding success of this methodology in the physical sciences has held out the hope for many years that a number of aspects of human affairs could similarly be studied using these procedures. Many people, however, maintained that mathematics had no role in these areas because of the presence of so many qualitative rather than quantitative factors. They believed that mathematical reasoning can be fruitfully employed only in domains where numbers, formulas, and clockwork regularities abound. This belief is fortunately not correct as we wish to explain briefly in what follows.

Some of this pessimism stems from a basic misunderstanding of the nature of mathematics, some from the usual prejudice that impedes relations between cultures and some from a fear instilled by Sunday supplement scientific propaganda frequently centering around computers. In this connection, let us note that it is sad that as a general rule mathematicians and scientists are far more familiar with the humanities than conversely.

In this paper we wish to emphasize the symbol manipulation capacity of the digital computer. We shall give some examples to illustrate this.

Finally, we wish to discuss kladistics, a theory connecting one event with another, a fundamental part of the field of history.

We shall give some references for further reading.

### 2. Mathematics

Let us describe mathematics as the study of conceptual structures, their transformations over time, and their interactions. If we replace the carefully chosen vague term "structure" by the equally vage term "system" and speak of the study of human systems over time, we obtain a reasonably good quick definition of the field of history.

Accepting the fact that any human activity must possess structure, indeed many different types of structures, it is plausible that any field can profit by the use of mathematical thinking. This was a popular idea during the Renaissance with great influence upon art, architecture and music. The interaction of mathematics with music goes back to the Greeks.

It is the function of the mathematician to discern and use this structure.

## 3. Classical Use of Mathematics

The classic use of mathematics is quite stylized. The structure (system, process, etc.) under consideration is first endowed with properties or qualities such as "position," "velocity" and so on which can be described in numerical terms. Some, but not all, of these can actually be measured. This modeling, as the activity is called, of course, requires intimate knowledge of the field and a great deal of trial and error.

No mathematical theories are intrinsic; all are superimposed. In order to appreciate the effort involved, it is essential to note that when we view existing approaches, we see only the successes. It is very difficult to estimate the ratio of successful to total attempts.

Next, certain rules are introduced to tell how these numerical quantities change over time. For example, there is the famous law of Newton, $F = ma$, force equals mass times acceleration. Acceleration is change of velocity; velocity is change of position. Using calculus, these statements translate into simple equations (usually differential equations) which can be applied by a scientist, or mathematician, to predict the future behavior of the system, i.e., to predict the future given the present, and sometimes to discover the past. This usually requires an enormous amount of arithmetic, whence the great significance of a digital or analog computer. This device has been responsible for two scientific revolutions, but this is a story in itself.

We have spoken in terms of equations, which means using an analytic language. Mathematics, however, has many languages at its disposal. A geometric, or topological language may be more useful and intuitive in many situations. Also, there is the technique of algebra. In describing structure, geometry and algebra will play a large role.

## 4. Basic Abilities of the Computer

When we use the term "computer" we are thinking of a device that possesses several remarkable properties. It can do arithmetic on a grand scale, it can store and retrieve numbers or data that can be translated into numerical symbols; it can follow instructions to perform these operations, and to display intermediate and final results. Thus, a digital computer can evaluate an arithmetic expression such as $1 \times 2 + 3 \times 4 + 5 \times 6 + 7 \times 8 + 9$ in the following steps:

$1 \times 2 = 2$; store 2
$3 \times 4 = 12$; add $2 + 12 = 14$; store 14
$5 \times 6 = 30$; add $30 + 14 = 44$; store 44
$7 \times 8 = 56$; add $56 + 44 = 100$; store 100
Add $100 + 9 = 109$; Answer.

The numerical solution of the equations of celestial mechanics require nothing more than this conceptually, given the laws of motion, and little more than this arithmetically.

Presumably, these facts should greatly influence the presentation of scientific theories and make widely available the scientific heritage bequeathed by such scientists as Archimedes, Galileo, Newton, Euler, Gauss. Unfortunately, those who have painfully acquired knowledge generally have little desire to share it, consistent with the medieval idea that "knowledge is power." This, however, is another digression.

Use of the computer is never routine. Although the basic ideas are simple, a great deal of skill is required to obtain numerical answers to numerical questions. One of the great advantages of the digital computer is that it never gets bored.

Before the development of the digital computer, it was impossible to treat the processes of the social sciences because of their complexities. There are so many interactions and variables to consider that a treatment by hand is impossible.

## 5. Symbol Manipulation

The foregoing arithmetical ability, important as it is, is only a special case of a more general skill of the computer, symbol manipulation. Thus, for example, the computer can deal with general nonnumerical symbols A, B, C, D, E, with the rules (algorithms): At each stage

1. A transforms into C
2. B into A
3. C into D
4. D into B
5. E into B.

We may then ask the question: If we start with the set of symbols ADE, what set will we have after 100 stages, after 1000 stages?

We can answer this by hand, of course: Referring to the transformation rules above, at the end of one stage, we have

$$\text{ADE} \rightarrow \text{CBB}$$

At the end of two stages,

$$\text{CBB} \rightarrow \text{DAA}$$

At the end of three stages

DAA $\rightarrow$ BCC

and so on. There are as might be expected more sophisticated approaches to questions of this type but this is of no matter here. These problems may be easily resolved by the use of matrix analysis.

The digital computer can carry through these operations in microseconds and either print out the final result, any desired intermediate results or the entire sequence. Alternatively, it can display these results on a computer screen as desired. The investigator can by pressing a few buttons ask questions such as:

"What would have happened if we started instead with ABE?"

"What would have happened if the transformation rule was changed slightly, i.e., A $\rightarrow$ D?"

"Could we ever end up with all A's, i.e., could B, C, D, and E ever disappear?"

The computer frees us from the burden of elementary operations and permits us to spend our time instead thinking up significant questions, interpreting results and improving models of real systems.

## 6. Chance Effects

The foregoing was an example of a deterministic process, much too rigid for most scientific and engineering purposes and certainly so for the social sciences. We could equally well, however, allow for chance effects, or in more mathematical terms, stochastic effects. Thus, we can allow rules of the following type.

1. There is a probability of 1/2 that A transforms into C, a probability of 1/2 that A becomes B.

2. There is a probability of 1/3 that B becomes C, D, or A,

and so on.

The computer in various ways can replicate this process, running thru it a large number of times to determine average behavior or extreme behavior. Generally it readily allows an examination of a huge set of possible patterns. This is often called a Monte Carlo procedure.

One of the advantages of a digital computer here is that it is very difficult for the human mind to follow all possibilities. There is a great tendency to overlook certain paths.

Again, many of these problems can be handled by matrix analysis.

## 7. Complexity

In many applications the transformation of a symbol depends upon the presence of other symbols. Thus, the rules (algorithms) may read:

> A transforms into C if B is present, otherwise into D, and so on.

We can combine this more complex algorithm with the stochastic behavior described above to obtain more realistic descriptions of processes.

It is essentially impossible for the human mind to perform an enumeration of cases systematically for a large number of stages in a process of this nature. The computer can, however, with the aid of simple programs (sets of computer instructions) carry out a thorough examination of cases and display the desired data in various ways.

It can, in addition, following another computer program interpret the results, select significant data, alter hypotheses to fit the observed facts (as a good scientist or historian might), add relevant factors, and so on. How far it can go in an analysis depends upon the trained and experienced human carrying out the investigation, i.e., upon the expert. Part of this training in expertise can now be furnished by computer studies of this nature, accelerated and motivated.

We can now study the fascinating "What if ..." questions of history, and explore countless hypotheses.

A description of the abilities of the digital computer will be found in

Bellman, R., Introduction to Artificial Intelligence, in process.

## 8. Music Generation

Viewed abstractly, we have a method of generating music by computer. This is one of the standard techniques that is used. We can do several things.

First of all, we can take a particular composer and analyze his style and generate the kind of music that he would.

Secondly, by the use of these methods, we can generate new music. Before the computer, this was done by Schillinger, see

Schillinger, Joseph, Schillinger System of Musical Composition, Carl Fischer, Inc., New York, 1946 (not in print);

Schillinger, Joseph, Mathematical Basis of the Arts, Johnson Reprint Corp., New York, 1948;

for Tin Pan Alley composers who were out of ideas. The method was also used by Mozart, see

W. A. Mozart, The Dice Composer, Koechel, 294D, A. Laszlo, ed.

## 9. Simulation

The reader versed in the classical uses of mathematics in science will realize that there is nothing conceptually new so far despite the absence of numbers.

We do begin to encounter conceptually novel processes when we do not allow the luxury of specific symbolic description, the existence of explicit rules of transformation, or the presence of a criterion for behavior. Nonetheless, we insist upon decision-making. This is typical of much political, economic, business and military decision-making. Although this kind of process can fruitfully be studied by simulation techniques, some new ideas are required which are discussed in another place, see

Bellman, R. and C. P. Smith, Simulation in Human Systems - Decision-Making in Psychotherapy, John Wiley & Sons, Inc., New York, 1973.

We have not discussed simulation here since it is discussed in other articles in the book.

## 10. A Simple Process

Suppose that we wish to study the process involved in a South American or African country changing from one form of government to another, say from a traditional military dictatorship to the traditional Communist dictatorship.

We might begin by listing several qualities:

A: strength of Catholic church

B: economic level

C: strength of military

D: climate

E: strength of middle class

F: strength of revolutionary movement.

In place of a single symbol, we might now use one, two or three. Thus, A denotes weak influence of Church, AA moderate influence, AAA strong influence; D denotes poor climate, DD moderate, DDD excellent, etc. Thus, we might write some rules: F $\rightarrow$ FF if B, DDD, and A or AA, and so on. Each country will possess its own descriptions and its own transformation rules. We can then ask for some long-term trends and predictions.

Similarly, we can study the evolution of certain legal concepts from the Magna Carta to Holmes, the decline of feudalism, the decline and fall of the Roman empire and in general the workings over time of any identifiable historical, economic and political forces on a particular system.

It is essential to note that the computer here is a logic machine, exploring the consequences of the data and hypotheses that the experts furnish. Different experts, different predictions.

## 11. Fuzzy Systems

We have been using the theory of fuzzy systems, the creation of Lotfi Zadeh. The theory of fuzzy systems enables us to handle qualitative variables. It is also another approach to uncertainty where classical probability is replaced by grade of membership.

One of the features of the theory of fuzzy systems is the use of linguistic variables, as we have done above.

There are many applications of the theory of fuzzy systems to history. For the way this theory can be used for decision-making, see

Bellman, R. and L. Zadeh, "Decision-Making in a Fuzzy Environment," Management Science, Vol. 17, 1970, pp. 141-164.

It is also true that many large systems possess their own logic, see

Bellman, R. and L. Zadeh, "Local and Fuzzy Logics," to be published.

Many other references will be found in these papers.

## 12. Kladistics

A basic question in structural theory in all fields of culture concerns the reconstruction of evolutionary or kladistic trees on pathways by inferences from the characteristics of organisms, systems, or data surviving at the present time. Let us cite the fields of biology and anthropology, the use of fossils in the case of archeology, and the domain of philology.

In recent years, methods have been developed for deducing trees which satisfy the condition of requiring a minimal number of evolutionary steps of changes in characters to explain the evolutionary history of the set of existing structures. See

Camin, J. H. and R. R. Sokal, "A Method for Deducing Branching Sequences in Phylogeny," Evolution, 19, pp. 311-326, 1965.

Hendrickson, J. A., "Clustering in Numerical Cladistics: A Minimum-Length Directed Tree Problem," Mathematical Biosciences, 3, pp. 371-381, 1968.

Kluge, A. G. and J. S. Farris, "Quantitative Phyletics and the Evolution of Anurans," Syst. Zool., 18, pp. 1-32, 1969.

Wagner, W. H., Jr., "Problems in the Classification of Ferns, in Recent Advances in Botany," University of Toronto Press, Toronto, pp. 841-844, 1961.

Estabrook, G. F., "A General Solution in Partial Orders for the Camin-Sokal Model in Phylogeny," Journal of Theoretical Biology, 21, pp. 421-438, 1968.

Sankoff, D., "Mathcing Sequences Under Deletion/Insertion Constraints," Proceedings of the National Academy of Science, U.S.A., 69, No. 1 pp. 4-6, 1972.

The principle of minimum evolution or "parsimony" is generally assumed in these papers as a suitable hypothesis in the absence of empirical laws of evolution. General algorithms for these "most parsimonious" trees have not been completely studied, although algorithms for close approximations called "Wagner trees" do exist. For ferns, see

Farris, J. S., "Methods for Computing Wagner Trees," Syst. Zool., Vol. 19, pp. 83-92, 1970.

Other conceptually related trees are studied in

Bellman, R., K. L. Cooke and J. Lockett, Algorithms, Graphs and Computers, Academic Press, Inc., 1970.

A very interesting algorithm for reconstructing phylogenetic relationships from protein amino acid sequence data under some restrictions about all distance measures is given in

Beyer, W. A., M. L. Stein, T. F. Smith and S. M. Ulam, "A Molecular Sequence Metric and Evolutionary Trees," Mathematical Biosciences, Vol. 17, 1973, pp. 444-461.

A source of references is

Bellman, R. and S. Dreyfus, Applied Dynamic Programming, Princeton University Press, 1962.

An excellent introduction to the subject where many further references will be found is in

Marchi, E. and R. I. C. Hansell, "Generalizations on the Parsimony Question in Evolution," Mathematical Biosciences, Vol. 17, No. 1/2, 1973, pp. 11-34.

## 13. Conclusion

What we have tried to do above is to sketch some of the ways in which mathematics can be used in history.

There are many applications of mathematics to history, and we have made no attempt to cover all of them.

What we wish to emphasize is how much opportunity exists.

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# REPORT DOCUMENTATION PAGE

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| 1. REPORT NUMBER<br>12494.11-M | 2. GOVT ACCESSION NO. | 3. RECIPIENT'S CATALOG NUMBER |
| 4. TITLE (and Subtitle)<br>Mathematics in the Field of History | | 5. TYPE OF REPORT & PERIOD COVERED<br>Technical Report |
| | | 6. PERFORMING ORG. REPORT NUMBER |
| 7. AUTHOR(s)<br>Richard Bellman | | 8. CONTRACT OR GRANT NUMBER(s)<br>DAHC04 76 G 0027<br>use DAAG29 |
| 9. PERFORMING ORGANIZATION NAME AND ADDRESS<br>University of Southern California<br>Los Angeles, California 90007 | | 10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS |
| 11. CONTROLLING OFFICE NAME AND ADDRESS<br>U. S. Army Research Office<br>Post Office Box 12211<br>Research Triangle Park, NC 27709 | | 12. REPORT DATE<br>April 1977 |
| | | 13. NUMBER OF PAGES<br>11 |
| 14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) | | 15. SECURITY CLASS. (of this report)<br>Unclassified |
| | | 15a. DECLASSIFICATION/DOWNGRADING SCHEDULE |

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

The findings in this report are not to be construed as an official Department of the Army position, unless so designated by other authorized documents.

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)


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